Abstract – The use of spherical, conical and hyperboloidal mirrors, combined with standard optics to allow 360° field of view for middle size robotic football league can degrade the image quality. This paper describes a simplified approach to calculate the resolution loss caused by the non-planar mirror, accounting for the influence of the various setup parameters on the resulting camera image.

I. INTRODUCTION

Mobile robots used in middle size robotic football league (Fig. 1) use a simple standard camera, pointed upwards, combined with a specially shaped mirror (spherical, conical, hyperboloidal, etc.). The simple approach of object localization in [1, 2] allows the computation of objects positions using only image data and setup parameters for objects at known height above the floor; however, image degradation due to the non-planar mirror shapes was not considered. This report describes an approach used to calculate image quality loss that can also point to the setup parameters that provide the best results.

The approach followed in [1, 2] uses angle invariance to compute positions of the objects in the field of view. The use of mirrors for a 360° field of view in a single image was described in [2]; various types of mirrors with surfaces such as cone, sphere and hyperboloid, all generated by the process of revolution over an axis, that should be aligned in the setup to coincide with the camera optical axis. For the image forming conditions in the setup, the pin-hole approach was
used. For the conical mirror, a virtual centre of projection was found for each vertical plane containing the camera optical axis. For the spherical and hyperboloidal mirrors the projection centre moved for each angle, although using a fixed point allowed results that were close enough for the application.

However, the visible image quality varied with the type of mirror in use, and the final shape (quasi-hyperboloidal) was obtained by trial and error. In this process we developed the approach of [1, 2] and this report refers the image quality conditions that could be aimed for the various types of mirror surfaces.

The pin-hole approach was not enough, and so a light cone model was used: we can define a cone supported on the lens iris contour line and with vertex on the object point. The lens produces the correspondent image cone, supported on the same iris contour line, but whose vertex is the corresponding image point on the sensor surface, as in Fig. 2. The axis of both cones corresponds to the ray-tracing model in the pin-hole approach. When a mirror is used, its reflective properties produce the changes also illustrated in Fig. 2. For each point, the mirror surface in use is only the small spot defined by the cone on the mirror. If it was a plane surface, just the angular deviation needed to be considered. But, for a non-planar mirror surface, the light rays near the cone surface suffer larger deviations compared to the central ray. Computing the cone spot size on the mirror surface and the corresponding maximum angular deviation from the plane (Fig. 3), a deviation-limit condition can be established in object terms and an image quality limiting condition established.

II. THE MIRROR SETUP

As sketched in Fig. 2, the pin-hole camera approach remains valid to establish the relationship between object and image points as far as a planar mirror is used. For the complex lens, the solid cone with vertex on the object point and pointed to the lens aperture contour is made to converge to the image point on the sensor surface by the lens system. The straight line linking the corresponding object and image points contains the optical projection centre that corresponds to the pin-hole approach. And this cone limit is kept with the use of a planar mirror.

In this way, the angular limits of the cone are determined by the distance between object and lens and the lens aperture in use. A smaller aperture and a larger object distance diminish the angle and also the mirror area in use for each image point.

The reflection of light rays of the cone is detailed in Fig. 3. If the mirror surface deviates from the plane, the reflection laws imply that the reflected ray deviates by an angle twice the surface angular deviation. This can be taken in consideration either for image forming rays (from object to image) or object localization (back propagation from image to object).

As it can be seen in Fig. 3, a small lateral deviation is also introduced to the angular deviation \(2\delta\), but considering the spot size, it can be discarded in a first order approximation. Considering the deviation \(2\delta\) the image forming light rays inside the cone can be provided not only from the theoretical single object point but also from a spot around it, limiting the image resolution that can be achieved.

III. THE EQUATIONS

As sketched in Fig. 2, the image forming cone originating from every visible object point travels the equivalent object lens distance, the first part before the reflection and the second part after it. The position of the virtual object point \(O'\) can be used to establish the ray-tracing diagram from the image point of view, without the mirror.

Considering the cylindrical symmetry of the setup, this analysis is done for a vertical plane that contains camera optical axis. For each visible object point, the corresponding image angle \(\alpha\) is transformed to an angle \((\pi/2-2\beta-\alpha)\) by the mirror using the spot centred at point \(R\), as sketched in Fig. 4 for a spherical mirror but valid for any mirror whose tilt at the point under consideration is \(\beta\).

For a lens of focal length \(f\) and using a numerical aperture \(n\) and for an object distance \(H\), the maximum plane angle between the cone axis and a surface ray can be put as (Fig. 2):

\[
\tan \theta = \frac{f}{2nH}.
\]
The cone spot size on the mirror can be computed from the iris diameter and the relative object-mirror distance to the lens-object distance (Fig. 2) and the angle of incidence (α+β). Dividing the object distance between $H_1$ and $H_2$, where $H_1$ is the distance lens-mirror and $H_2$ the distance mirror-object:

$$H = H_1 + H_2, \quad H_1 \ll H_2.$$  

(2)

Considering that the lens to mirror-spot distance is much smaller than the lens-object distance, the iris projection can be considered as the spot size on the mirror for practical situations. Once defined the area of the mirror in use, the maximum tilt of the surface spot relative to the centre point can be used to calculate the angular tilt relative to the centre, and it depends only on the mirror shape. We used two orthogonal directions tangent to the mirror surface on point $R$, one on the vertical plane represented in Fig. 4, and the other orthogonal to this plane. The influence of non-plane mirror can be decomposed for these two orientations, that is, two deviations, one on the main vertical plane and the other corresponding to an horizontal deviation.

The spot size of the cone on the mirror surface can be described by their maximum horizontal and vertical half-dimensions $S_H$ and $S_V$:

$$S_H = \frac{H_2}{2} \tan(\theta)$$  

(3)

and, considering (α+β) the mirror surface tilt relative to light cone axis (as in Fig. 4, from [2]):

$$S_V = \frac{H_2}{2} \tan(\theta) \frac{\cos(\alpha + \beta)}.$$  

(4)

The light ray deviations are twice the values in (7) and (8), as we choose to keep the definitions in Fig. 3.

In object terms, angular deviations introduced by the mirror correspond to areas on the object that can correspond to an image point and forbidding better resolution that the size of those areas on the object. From the deviation angles 2$\delta_H$ and 2$\delta_V$ at distance $H_2$, the horizontal and vertical object half-spots sizes are respectively

$$SO_H = \frac{f H_2}{n H r}$$  

(9)

and

$$SO_V = \frac{f H_2}{n H R \cos(\alpha + \beta)}.$$  

(10)

This can be simplified considering $H_2/H = 1$. And if put in terms of the iris diameter $f/n$ and considering the full spot, that is twice the results in (9) and (10) we can define

$$SO_H = \frac{2 H_2}{r} \frac{f}{n}$$  

(11)

$$SO_V = \frac{2 H_2}{R \cos(\alpha + \beta)} \frac{f}{n}.$$  

(12)

In resume, the system resolution in object terms can be put as the iris diameter amplified by a factor depending on the ratio object distance over a mirror related dimension.

For the spherical mirror, the arc of circle defined by the vertical half-spot size corresponds to the tilt angle, but, as the angle of incidence increases towards the border of the mirror, the constant aperture image forming cone uses a larger spot on the mirror and so corresponds to a larger mirror relative tilt and also a larger deviation, where $R$ is the ray of the spherical mirror:

$$\delta_t = \frac{S_v}{R}.$$  

(6)

This equation also translates the conical mirror case: just considering $R = \infty$ nulls the angular deviation, as considered above.

Combining (1), (3) and (5)

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(7)

And, from (1), (4) and (6),

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In instance, as in the case of image in Fig. 5, the amplification, in horizontal terms, can be quite large: for an iris of 1 mm and using a mirror with a diameter of 12 cm, a point using the middle region ($r = 3$ cm) and corresponding to an object distance of approximately 2 meters, would be around 7 cm.
But for a common lens focal distance of 4mm, at 2m, the ratio image/object is 1:500, and the 7 cm size object corresponds to an image of 14 μm that is in the order of the pixel size of the camera. This means that the camera in normal conditions (without the mirror system) could barely distinguish the blurred spot on the object.

IV. CONCLUSIONS

The hyperboloidal mirror behaves as a conical mirror for large values of α, without vertical deviation; for small values of α it approximates the spherical mirror case.

The choice of a hyperboloidal or quasi-hyperboloidal solves the problem of the simple conic mirror: in the image optical centre we obtain the image of a circumference corresponding to a cone surface of angle β around the robot and outer surroundings are severely distorted. Smoothing the cone apex to a rounded surface (approximately spherical or hyperboloidal) spreads the image information for a larger area (equivalent to use lower β values). The results can be observed in the image in Fig. 5. The shadow in the center represents the plastic cone, painted black, and glued on the mirror apex, that is used to adjust camera-mirror distance (in this case the is a slight misalignment with the camera lens).

Also for the conical mirror, for lower values of α (image points closer to the image centre), the horizontal angle spread δH increases. But for this situation, a low resolution in object terms was to be expected, as pixel dimensions limit the possible resolution.

The horizontal resolution (in image corresponds to the sagittal resolution) is somewhat reduced for low values of α, but for objects at larger distances it improves.

The results for the use of the spherical mirror present the opposite effect: for lower angles the spread is minimal (corresponding to an approximation to the plane mirror case); for larger angles, the combination with larger increase of angle β compresses radially the image resulting angles, producing lower quality images.

For the hyperboloidal mirror, tilt tends to zero for the center spot and tends to the conical case for the outer zone; the same conclusions as for the conical shape mirror apply.

The spherical mirror is a different case. The use of the central zone presents no problem (tilt around zero). But the borders of this mirror correspond to an increase of the tilt that produces a larger spot (on the mirror) and the corresponding larger variation of the surface tilt from the center to the border of the spot.

This means that, unless the use of spherical mirrors is limited to a central region, the image is severely distorted and fuzzy towards the borders. The increase of the viewing angle that spherical mirrors allow is limited by image quality.

Camera sensor layout, using square pixels, also present resolution limits to objects that produce images close to the centre of the image. The degradation introduced by the use of the mirror cannot be blamed for all the problems. The polar representation on the image that the 360° field of view imposes and the square array of pixels in common use are another problem, to be solved possibly by a different pixel arrangement on the sensor.

Although the image quality can theoretically be impaired in some cases, the use of mirrors to increase the angle of view of a single standard camera is common practice in this sort of application.

The use of spherical mirrors is practically none today for these robotic applications. Its substitution for hyperboloidal or quasi-hyperboloidal surfaces, although more difficult to manufacture, is common practice in this field of application.

Conical mirrors, much simpler to manufacture are usually disregarded, due to the central dead spot. Turning the cone vertex into a round, soft shape avoids the dead-zone syndrome, but implies a time-consuming calibration for distance assessment of objects in the central area.

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References

